Theory of acoustic analog of magneto-optic polar Kerr effect under magnon-phonon resonance

A. M. Burkhanov, K. B. Vlasov, V. V. Gudkov, and B. V. Tarasov

February 2, 2008

1 Introduction

The acoustic analog of magneto-optic Kerr effect consists of variation of the polarization of the elastic wave after its reflection from an interface between magnetic medium and isotropic non-magnetic one. Quantitative characteristics of the effects are polarization parameters: ε – the ellipticity which modulus is the ratio of the minor and major ellipse axes and ϕ – the angle of rotation of the polarization plane or, more correctly, of the major ellipse axis if $\varepsilon \neq 0$.

There are three possible variants according to directions of the static magnetic induction $\mathbf{B}$, plane of incidence of the wave and the unit vector $\mathbf{q}$ normal to the interface plane: polar ($\mathbf{B} \parallel \mathbf{q}$); meridional (or longitudinal) ($\mathbf{q} \times \mathbf{k} \perp \mathbf{B}$); and equatorial (or transverse) ($\mathbf{q} \times \mathbf{k} \parallel \mathbf{B}$). The incident waves are called p-type if the elastic displacement vector $\mathbf{u}$ is perpendicular to $\mathbf{q} \times \mathbf{k}$, and s-type if $\mathbf{u}$ is parallel to this vector.

The acoustic analog of the Kerr effect was predicted by Vlasov and Kuleev [1]. Theory of the effect for an inclined incidence of a wave was developed by Vlasov and Babushkin [2]. In this paper an isotropic magnetic medium (at $\mathbf{B} = 0$) was discussed.

Here we consider the following model. The medium I is a semi-infinite isotropic non-magnetic and non-dissipative while the medium II is a ferromagnetic one with cubic lattice and easy axis along [111]-type crystallographic direction. The interface is perpendicular to the [001] axis and the plane of incidence is given by the equation $x = 0$. The medium II has a form of a sphere with a small (respectively the radius of the sphere) plane area contacting with the medium I.

In this case the elastic displacements of the reflected shear waves $\mathbf{u}'$ may be represented by a sum of mutually orthogonal vectors $\mathbf{u}'_x$ and $\mathbf{u}'_\perp$, where

$$\mathbf{u}'_x = u'_x \mathbf{e}_x \qquad \text{and} \qquad \mathbf{u}'_\perp = u'_\perp \mathbf{e}_\perp, \tag{1}$$

and $\mathbf{e}_x$ and $\mathbf{e}_\perp$ are unit vectors. By introducing reflection coefficients $R_n^\pm$ for circular components defined by $R_n^\pm \equiv u'_x/u_n \pm i u'_\perp/u_n$ and expressing them in the form $R_n^\pm = |R_n^\pm| \exp(i\rho_n^\pm)$, we obtain expressions for the ellipticity and the rotation of the polarization of an elastic wave of n-type upon reflection from the interface as

$$\varepsilon = \frac{|R_n^+| - |R_n^-|}{|R_n^+| + |R_n^-|} \qquad \text{and} \qquad \phi = \frac{1}{2}\left(\rho_n^- - \rho_n^+\right). \tag{2}$$

In definition (1) and later on a symbol with a prime relates to characteristics of a reflected wave, with two primes – to refracted (transmitted to the second medium) and without primes – to the incident wave; n may refer to shear mode of arbitrary linear polarization.

2   Boundary conditions

We will consider classical approach given in Ref. 3 and generalize it to the case of magnetoelastic interaction in one of the contacting media. Thus, the boundary conditions require continuity of the displacements at the interface, and of the force acting on the interface:

$$\mathbf{u}_n + \sum_{i=1}^{3} \mathbf{u}'_i - \sum_{j=1}^{m} \mathbf{u}''_j = 0, \tag{3}$$

$$(\tau_{mz})_n + \sum_{i=1}^{3} (\tau_{mz})'_i - \sum_{j=1}^{m} (\tau_{mz})''_j = 0, \tag{4}$$

The sums in Eqs. (3)–(4) contain contributions of all the normal (eigen) modes that have elastic displacements. In harmonic approximation the boundary conditions can be presented as ones for complex amplitudes and phases. The last one leads to the following conclusions: frequences of all the waves (incident, reflected, and refracted) are equal, all the wave vectors belong to a common plane, namely, the plane of incidence, and, besides,

$$\frac{\sin\theta}{s} = \frac{\sin\theta'_i}{s'_i} = \frac{\sin\theta''_j}{s''_j}, \tag{5}$$

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where θ is the angle of incidence, θ_i' that of reflection, corresponding to the wave with phase velocity s_i', and θ_j'' that of refraction, corresponding to the wave with phase velocity s_j''.

The boundary conditions given by Eqs. (3)–(4) should be appended with ones for variable magnetization those were formulated in Ref. 1.

Remind: all the eigen modes that have elastic displacements should be represented in boundary conditions and the tensions due to the magnetoelastic interaction should be in Eq. (4).

There are three such solutions for an isotropic dielectric medium: one longitudinal and two degenerate transverse.

For the magnetic medium, in general, there should be five solutions. However, here we will consider a weak-coupling approximation. In this approximation the interaction between the subsystems results in small variation of the dispersion curves of normal modes which become coupled phonon-magnon (or phonon-like) and magnon-phonon (or magnon-like) ones. Only three elastic-like modes can be accounted in Eqs. (3)–(4) and boundary conditions for magnetization are not used for obtaining $R_n^{\pm}$.

3 System of equations for determination of normal modes

To obtain the wave vectors and complex amplitudes of the eigen modes it is necessary to solve the equations of the elasticity theory for the first medium and for the second one – the system consiting of Maxwell's equations, equation of motion for magnetization, and equations of elasticity theory. We propose that the crystal is in magnetically saturated state and only small variations of magnetization can occur.

Thus, for the first medium we have

$$\rho_1 \ddot{u}_i = \frac{\partial \tau_{ij}}{\partial x_j}, \tag{6}$$

with tensions τ_{ij} defined by

$$\tau_{ij} = \frac{\partial W}{\partial \varepsilon_{ij}}, \tag{7}$$

where ρ_1 is the density of the medium I, $\varepsilon_{ij} = \frac{1}{2}\left(\partial u_i/\partial x_j + \partial u_j/\partial x_i\right)$ are components of strain (or deformation) tensor, and the density of free energy W contains only the elastic energy W_L.

For the second medium we have equation of motion for volume element similar to (6). However, the free energy which defines the tensions and effec-

tive field $\mathbf{H}_e$ should be presented by elastic, magnetic, and magnetoelastic terms.

Formally, we should use the superscript ″ on all of the parameters and variables related to the magnetic medium, but we will defer this until such terms for all of the waves, incident, reflected, and transmitted, appear in the same expression.

It is preferable to write the equation of motion for magnetization in the form of the Gilbert equation:

$$\dot{\mathbf{M}} = \gamma \mathbf{M} \times \mathbf{H}_e + \frac{\alpha_0}{M_0} \mathbf{M} \times \dot{\mathbf{M}}, \tag{8}$$

where α_0 is the relaxation parameter, M_0 is the modulus of static magnetization in homogeous state, and $\mathbf{M}$ is total magnetization.

The density of total free energy of a magnetic crystal in addition to elastic and magnetoelastic energy, contains Zeeman energy W_0, exchange energy W_{ex}, energy of magnetic anisotropy W_a, and magnetostatic energy W_d. The last two terms determines the field of magnetic anisotropy $\mathbf{H}_a$ and the static $\mathbf{H}_d^0$ and variable $\mathbf{h}$ demagnetizing fields. We will use demagnetizing factor corresponding to a spherical specimen considering the plane areas as small. Expressions for the types of energy and fields can be found in Ref. 4.

Since we are limiting ourselves to the case where all wave vectors lie in the $y - z$ plane, the complete system has the form

$$
\begin{aligned}
a_{11}u_1 + a_{14}m_1 &= 0, \\
a_{22}u_2 + a_{23}u_3 + a_{25}m_2 &= 0, \\
a_{32}u_2 + a_{33}u_3 + a_{35}m_2 &= 0, \\
a_{42}u_2 + a_{43}u_3 + a_{44}m_1 + a_{45}m_2 &= 0, \\
a_{51}u_1 + a_{54}m_1 + a_{55}m_2 &= 0,
\end{aligned}
\tag{9}
$$

where m_i are components of variable magnetization,

$$a_{11} = \rho\omega^2 - c_{1313}(k_2^2 + k_3^2), \quad a_{22} = \rho\omega^2 - c_{1111}k_2^2 - c_{1313}k_3^2,$$

$$a_{33} = \rho\omega^2 - c_{1111}k_3^2 - c_{1313}k_2^2, \quad a_{44} = a_{55} = i\omega,$$

$$a_{14} = a_{25} = -\frac{ib_2 k_3}{M_0}, \quad a_{23} = a_{32} = -(c_{1122} + c_{1313})k_2 k_3,$$

$$a_{35} = -\frac{ib_2 k_2}{M_0}, \quad a_{42} = -a_{51} = i\gamma b_2 k_3,$$

$$a_{43} = i\gamma b_2 k_2, \quad a_{54} = \gamma\left[\frac{2A}{M_0}(k_3^2 + k_2^2) + H_i\right] - i\alpha_0\omega,$$

$$a_{45} = -\gamma\left[\frac{2A}{M_0}(k_3^2 + k_2^2) + H_i + \frac{4\pi k_2^2 M_0}{k_2^2 + k_3^2}\right] + i\alpha_0\omega,$$

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ρ is the density of the medium II, $H_i = H + H_d^0 + H_a$, c_{ijkl} are elastic moduli, b_2 is magnetoelastic constant. It was assumed that all variables are proportional to $\exp\left[i\left(\omega t - \mathbf{k} \cdot \mathbf{r}\right)\right]$.

Existing of non-trivial solutions of the system (9) requires its determinant to be equal to the zero. It is dispersion equation since it gives expressions for the wave vectors of eigen modes. As the angle of incidence is given and therefore, according to Eq. (5), $\mathrm{Re}(k_2)$ for all of the eigenvectors is defined, the unknown values are $\mathrm{Re}(k_3)$, $\mathrm{Im}(k_3)$, and $\mathrm{Im}(k_2)$.

4   Solution of dispersion equation

Using the last two equations of (9), we may express m_1 and m_2 as functions of u_1, u_2, and u_3:

$$m_i = b_{ij}u_j, \tag{10}$$

substitute these expressions into the first three equations, and set the determinant of the new system (containing these three equations) equal to zero:

$$
\begin{aligned}
(a_{11} &+ a_{14}b_{11}) \\
&\times \left[(a_{22} + a_{25}b_{22})(a_{33} + a_{35}b_{23}) - (a_{32} + a_{35}b_{22})(a_{23} + a_{25}b_{23})\right] \\
&- a_{14}b_{12}\left[a_{25}b_{21}(a_{33} + a_{35}b_{23}) - a_{35}b_{21}(a_{23} + a_{25}b_{23})\right] \\
&+ a_{14}b_{13}\left[a_{25}b_{21}(a_{32} + a_{35}b_{22}) - a_{35}b_{21}(a_{22} + a_{25}b_{22})\right] = 0,
\end{aligned}
\tag{11}
$$

where

$$
\begin{aligned}
b_{11} &= \frac{a_{45}a_{51}}{a_{44}a_{55} - a_{54}a_{45}}, \quad b_{12} = \frac{-a_{55}a_{42}}{a_{44}a_{55} - a_{54}a_{45}}, \\
b_{13} &= \frac{-a_{55}a_{43}}{a_{44}a_{55} - a_{54}a_{45}}, \quad b_{21} = \frac{-a_{51}a_{44}}{a_{44}a_{55} - a_{54}a_{45}}, \\
b_{22} &= \frac{a_{54}a_{42}}{a_{44}a_{55} - a_{54}a_{45}}, \quad b_{23} = \frac{a_{43}a_{54}}{a_{44}a_{55} - a_{54}a_{45}},
\end{aligned}
\tag{12}
$$

We will solve the dispersion equation (11) by an iterative method. The zero order approximation will be obtained by letting the elements describing the magnon-phonon interaction (i.e., a_{14}, a_{25}, a_{35}, a_{41}, a_{42}, a_{43}, a_{51}, a_{52}, and a_{53}) vanish. One of the solutions is

$$k_3^2 = \rho\omega^2 c_{1313}^{-1} - k_2^2, \tag{13}$$

whereas the next two are

$$k_3^2 = -P \pm \left(P^2 - Q\right)^{1/2}, \tag{14}$$

where

$$P = \left[\left(c_{1111}^2 + c_{1313}^2 - (c_{1122} + c_{1313})^2 \right) k_2^2 \right.$$
$$\left. - \rho\omega^2 \left(c_{1313} + c_{1111} \right) \right] \left(2c_{1111}c_{1313} \right)^{-1},$$
$$Q = \rho^2\omega^4 - \rho\omega^2 k_2^2 \left(c_{1313} + c_{1111} \right) + c_{1313}c_{1111} k_2^4,$$

Eqs. (13)–(14) for k_3 relate to pure elastic waves: transverse s-type mode with wave vector $\mathbf{k}_s$, quasi-transverse (quasi-p) mode with wave vector $\mathbf{k}_p$, and quasi-longitudinal (quasi-l) mode with wave vector $\mathbf{k}_l$. At this point we have obtained real wave vectors (i.e., their imaginary parts are zero), since energy dissipation is considered to be in the magnetic subsystem only.

Next, the resulting expressions for k_3 should be substituted in turn into the equations which describe the magnetic subsystem and the magnon-phonon interaction. Recall that all k_2 in the zeroth approximation are real, equal, and known. Thus the solution for k_3 relating to the first weakly-coupled phonon-magnon mode (now of the quasi-s type, since it has y and z components of the elastic displacement) is

$$(k_3)_{qs}^2 = -k_2^2 + \frac{1}{c_{1313}} \left[\rho\omega^2 + a_{14}b_{11} + [(a_{22} + a_{25}b_{22})(a_{33} + a_{35}b_{23}) \right.$$
$$- (a_{32} + a_{35}b_{22})(a_{23} + a_{25}b_{23})]^{-1}$$
$$\times [a_{14}b_{13}[a_{25}b_{21}(a_{32} + a_{35}b_{22}) - a_{35}b_{21}(a_{22} + a_{25}b_{22})]$$
$$\left. - a_{14}b_{12}[a_{25}b_{21}(a_{33} + a_{35}b_{23}) - a_{35}b_{21}(a_{23} + a_{25}b_{23})]]\right], \quad (15)$$

where all a_{ij} and b_{mn} are functions of $(k_3)_s$.

The other two solutions relating to the coupled phonon-like modes are written in the form of (14), however the parameter Q should now be given by

$$Q = \rho^2\omega^4 - \rho\omega^2 \left[k_2^2 (c_{1313} + c_{1111}) \right] + c_{1313}c_{1111}k_2^4 + a_{35}a_{22}b_{23} + a_{25}a_{33}b_{22}$$
$$- a_{32}a_{25}b_{23} - a_{35}a_{23}b_{22} - a_{14}[a_{11} + a_{14}b_{11}]^{-1}$$
$$\times [b_{12}[a_{25}b_{21}(a_{33} + a_{35}b_{23}) - a_{35}b_{21}(a_{23} + a_{25}b_{23})]$$
$$- b_{13}[a_{25}b_{21}(a_{32} + a_{35}b_{22}) - a_{35}b_{21}(a_{22} + a_{25}b_{22})]]. \quad (16)$$

Note that the solution of Eq. (14) with the $+$ sign before the root describes the coupled quasi-p mode for which all a_{ij} and b_{kn} in Eq. (16) are functions of $(k_3)_p$, whereas that with the $-$ sign describes the coupled quasi-l mode for which all a_{ij} and b_{kn} are functions of $(k_3)_l$.

Having obtained the complex z components of the wave vectors, we can now determine the imaginary part of the y components for a particular $\mathbf{k} =$

$k\mathbf{e_k}$, where k is a complex quantity and $\mathbf{e_k}$ is a unit vector in the $\mathbf{k}$ direction. Thus, the definition $\mathrm{Re}(k_i) = \mathrm{Re}[k\cos(\mathbf{k} \cdot \mathbf{e}_i)]$ should also be correct for the imaginary parts of the wave vector components. For every $\mathbf{k}_{qj}$ it follows that

$$\mathrm{Im}(k_2)_{qj} = \mathrm{Im}(k_3)_{qj}\frac{\cos(\mathbf{k} \cdot \mathbf{e}_3)}{\cos(\mathbf{k} \cdot \mathbf{e}_2)} = \mathrm{Im}(k_3)_{qj}\frac{\mathrm{Re}(k_2)_{qj}}{\mathrm{Re}(k_3)_{qj}},$$

where the index q stands for *quasi-* and j for s, p, or l.

To illustrate the differences in the positions of magnon-phonon resonances for waves of different polarizations and to compare them with data for $\theta = 0$, Fig. 1 shows the resonant frequency ω_r versus magnetic field calculated for a YIG single crystal. This frequency is determined by requiring that the real part of the resonant denominator $a_{44}a_{55} - a_{54}a_{45}$ in the definitions of b_{ij} given by expresions (12) vanish. Recall that θ is the angle of incidence, while the angle of refraction for $\theta \neq 0$ depends on H at a particular frequency and therefore cannot be a fixed property of curves B–D. It can be seen that in general one can observe manifestation of three resonances in the properties of a reflected wave.

5   Complex amplidutes of vibrations and tensions

The complex amplitudes of the elastic vibrations and tensions (elastic and magnetoelastic) can be written by inserting the eigen wave vectors into the system (9). In both cases magnetization is expressed in terms of u_1, u_2, and u_3 according to Eq. (10) .

The first three equations of (9), written in the form

$$c_{mn}u_n = 0 \quad (m, n = 1, 2, 3) \tag{17}$$

yield complex amplitudes expressed in terms of those which do not vanish for $b_2 \to 0$ and $\theta \to 0$ (i.e., in terms of U''_{1s} for quasi-s mode, U''_{2p} for quasi-p, and U''_{3l} for quasi-l; we omit the q index here and later on for complex amplitudes and wave vectors in magnetic medium), $c_{11} = a_{11} + a_{14}b_{11}, c_{12} = a_{14}b_{12}, c_{13} = a_{14}b_{13}, c_{21} = a_{25}b_{21}, c_{22} = a_{22} + a_{25}b_{22}, c_{23} = a_{23} + a_{25}b_{33}, c_{31} = a_{35}b_{21}, c_{32} = a_{32} + a_{35}b_{22}, c_{23} = a_{33} + a_{35}b_{33}$, b_{ij} are defined by (12).

So in the magnetic medium we have the following expressions for the complex amplitudes of phonon-like quasi-s, -p, and -l modes where we denote their wave vectors by $\mathbf{k}''_s$, $\mathbf{k}''_p$, and $\mathbf{k}''_l$:

$$U''_{2s} = \frac{c_{31}c_{23} - c_{21}c_{33}}{c_{22}c_{33} - c_{32}c_{23}}U''_{1s} \equiv f_1(\mathbf{k}''_s)U''_{1s} \equiv A_{21}U''_{1s},$$

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Figure 1: Magnetic field dependences of the resonant frequency for waves propagating in a YIG single crystal with $\mathbf{H}$ parallel to [001]. Curve A corresponds to a transverse wave propagating along $\mathbf{H}$ (normal incidence at the interface), curves B, C, and D – to quasi-s, quasi-p, and quasi-l modes, respectively, calculated for the angle of incidence $\theta = 22.5°$.

$$U''_{3s} = \frac{c_{32}c_{21} - c_{22}c_{31}}{c_{22}c_{33} - c_{32}c_{23}} U''_{1s} \equiv f_2(\mathbf{k}''_s)U''_{1s} \equiv A_{31}U''_{1s},$$

$$U''_{1p} = \frac{c_{32}c_{13} - c_{12}c_{33}}{c_{11}c_{33} - c_{31}c_{13}} U''_{2p} \equiv f_3(\mathbf{k}''_p)U''_{2p} \equiv A_{12}U''_{2p},$$

$$U''_{3p} = \frac{c_{31}c_{12} - c_{11}c_{32}}{c_{11}c_{33} - c_{31}c_{13}} U''_{2p} \equiv f_4(\mathbf{k}''_p)U''_{2p} \equiv A_{32}U''_{2p}, \tag{18}$$

$$U''_{1l} = \frac{c_{23}c_{12} - c_{13}c_{22}}{c_{11}c_{22} - c_{21}c_{12}} U''_{2p} \equiv f_5(\mathbf{k}''_l)U''_{3l} \equiv A_{13}U''_{3l},$$

$$U''_{2l} = \frac{c_{21}c_{13} - c_{11}c_{23}}{c_{11}c_{22} - c_{21}c_{12}} U''_{2p} \equiv f_6(\mathbf{k}''_l)U''_{3l} \equiv A_{23}U''_{3l}.$$

In the non-magnetic medium we have an incident wave of p-type and reflected waves of s-, p-, and l-types with the angles of reflection defined by Eq. (5).

The complex amplitudes of the tensions associated with quasi-s, quasi-p, and quasi-l waves can be written in the form

$$(\tau^0_{13})''_s = \left[-ic_{1313}(k''_3)_s + \frac{b_2}{M_0}(b_{11} + b_{12}A_{21} + b_{13}A_{31}) \right] U''_{1s} \equiv (T_{13})''_s U''_{1s},$$

$$(\tau^0_{23})''_s = \left\{ -ic_{2323}\left[(k''_2)_s A_{31} + (k''_3)_s A_{21} \right] \right.$$
$$\left. + \frac{b_2}{M_0}(b_{21} + b_{22}A_{21} + b_{23}A_{31}) \right\} U''_{1s} \equiv (T_{23})''_s U''_{1s},$$

$$(\tau^0_{33})''_s = -i\left[c_{3322}(k''_2)_s A_{21} + c_{3333}(k''_3)_s A_{31} \right] U''_{1s} \equiv (T_{33})''_s U''_{1s},$$

$$(\tau^0_{13})''_p = \left[-ic_{1313}(k''_3)_p A_{21} + \frac{b_2}{M_0}(b_{11}A_{12} + b_{12} + b_{13}A_{32}) \right] U''_{2p}$$
$$\equiv (T_{13})''_p U''_{2p},$$

$$(\tau^0_{23})''_p = \left\{ -ic_{2323}\left[(k''_2)_p A_{32} + (k''_3)_p \right] + \frac{b_2}{M_0}(b_{21}A_{12} + b_{22} + b_{23}A_{32}) \right\} U''_{2p}$$
$$\equiv (T_{23})''_p U''_{2p},$$

$$(\tau^0_{33})''_p = -i\left[c_{3322}(k''_2)_p + c_{3333}(k''_3)_p A_{32} \right] U''_{2p}$$
$$\equiv (T_{33})''_p U''_{2p},$$

$$(\tau^0_{13})''_l = \left[-ic_{1313}(k''_3)_l A_{13} + \frac{b_2}{M_0}(b_{11}A_{13} + b_{12}A_{23} + b_{13}) \right] U''_{3l}$$
$$\equiv (T_{13})''_l U''_{3l},$$

$$(\tau^0_{23})''_l = \left\{ -ic_{2323}\left[(k''_2)_l + (k''_3)_l A_{23} \right] + \frac{b_2}{M_0}(b_{21}A_{13} + b_{22}A_{23} + b_{23}) \right\} U''_{3l}$$

$$\equiv (T_{23})''_l U''_{3l},$$

$$(\tau^0_{33})''_l = -i\left[c_{3322}(k''_2)_l A_{23} + c_{3333}(k''_3)_l\right] U''_{3l} \equiv (T_{33})''_l U''_{3l}.$$

$$(19)$$

Note that all of the coefficients $(T_{ab})''_j$ are functions of $\mathbf{k}''_j$, where j indicates the type of the wave (quasi-s, -p, or -l) in the magnetic medium.

The tensions in the non-magnetic medium may be written in terms of the Lamé constants λ and μ .

6 Ellipticity and rotation of the polarization

To obtain the amplitudes of the reflected waves, we have to write Eqs. (3)–(4) in the form:

$$\frac{U''_{1s}}{U_p} + A_{12}\frac{U''_{2p}}{U_p} + A_{13}\frac{U''_{3l}}{U_p} - \frac{U'_p}{U_p} = 0$$

$$A_{21}\frac{U''_{1s}}{U_p} + \frac{U''_{2p}}{U_p} + A_{23}\frac{U''_{3l}}{U_p} - \cos\theta\,\frac{U'_p}{U_p} + \sin\theta'_l\,U'_l/U_p = \cos\theta$$

$$A_{31}\frac{U''_{1s}}{U_p} + A_{32}\frac{U''_{2p}}{U_p} + \frac{U''_{3l}}{U_p} - \sin\theta\,\frac{U'_p}{U_p} - \cos\theta'_l\,\frac{U'_l}{U_p} = -\sin\theta$$

$$(T_{13})''_s\frac{U''_{1s}}{U_p} + (T_{13})''_p\frac{U''_{2p}}{U_p} + (T_{13})''_l\frac{U''_{3l}}{U_p} - i\mu\frac{\omega}{s_p}\cos\theta\,\frac{U'_s}{U_p} = 0 \qquad (20)$$

$$(T_{23})''_s\frac{U''_{1s}}{U_p} + (T_{23})''_p\frac{U''_{2p}}{U_p} + (T_{23})''_l\frac{U''_{3l}}{U_p}$$

$$-i\mu\frac{\omega}{s_p}\cos 2\theta\,\frac{U'_p}{U_p} + i\mu\frac{\omega}{s_l}\sin 2\theta'_l\,\frac{U'_l}{U_p} = -i\mu\frac{\omega}{s_p}\cos 2\theta$$

$$(T_{33})''_s\frac{U''_{1s}}{U_p} + (T_{33})''_p\frac{U''_{2p}}{U_p} + (T_{33})''_l\frac{U''_{3l}}{U_p} - 2\,i\mu\frac{\omega}{s_p}\sin 2\theta\,\frac{U'_p}{U_p}$$

$$-\left[2\,i\frac{\omega}{s_l}\left(\lambda + 2\mu\cos^2\theta'_l\right)\right]U'_l/U_p = 2\,i\mu\frac{\omega}{s_p}\sin 2\theta$$

Finally, we have a system of six inhomogeneous linear equations in six unknowns. Our particular interest is in U'_s/U_p and U'_p/U_p (note U'_s and U'_p are complex amplitudes of u'_x and $u'_\perp$, respectively, in the definition (1)), which yield the reflection coefficients in terms of complex amplitudes $R^\pm_p = U'_s/U_p \pm i\,U'_p/U_p$.

Thus, after solving the system (20), we have obtained all the data required to determine ε and ϕ from Eqs. (2).

To illustrate the phenomena, Fig. 2 shows the Kerr rotation and ellipticity which occur at a quartz-YIG interface. Calculations performed on the basis of the scheme given in section 4 show that variations of the wave vector related to a quasi-s type mode exceed those of quasi-p and quasi-l modes by factors of 10^2 and 10^3, respectively. As a consequence, only the z component of $\Delta\mathbf{k}_s''$ is shown in this figure to compare with the behaviors of $\varepsilon(H)$ and $\phi(H)$. Remember we assumed that the ferromagnet is in a magnetically homogeneous state, thus the results are valid only above the field of magnetic saturation, $H_S = 667$ Oe. In Fig. 2 (b), (c), (e), and (f) these parts of the curves are shown as solid lines. It follows that only the high field wing of the curves can be observed at the frequency of 53.5 MHz with the relaxation parameter $\alpha_0 > 0.1$.

Acknowledgement

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Figure 2: (a), (d) Magnetic field dependences of the real (A) and imaginary (B) parts of the z component of the wave vector of a quasi-s phonon-like mode with a frequency of 53.5 MHz propagating in a YIG single crystal for an angle of incidence $\theta = 22.5°$ and rotation of the polarization (b), (e) and the ellipticity (c), (f) of the reflected wave calculated with $\alpha_0 = 0.5$ for (a)–(c) and 0.1 for (d)–(f); $\Delta \mathbf{k}_s \equiv \mathbf{k}_s''(H) - \mathbf{k}_s''(0)$.